Are You Confused On How to Calculate Decibels

What is a logarithm ?
The simplest definition of a logarithm is that of numerical result of a number raised to some power
For example 10 x 10 = 100 can also be written as 10^2 or $Log_{10}(100)$
Note that in the statement "Log_{10}" the base is stated with the number "10" this defines that the log is using powers of 10.

One can have any base number one wishes, but mathematically we only use two kind of powers base ten (Log) and the natural logarithm of base "2.718281828459" noted as (Ln)

We are only concerned with the base 10 logarithms (Log)
Since a logarithm is the result of numbers based on "10" raised to some power, it makes it easy to combine the results as if we multiplied the numbers.

Example $10^2 \times 10^2 = 10^4$
What I have done is to simply add the exponents $10^{(2+2)} = 10^4$
So let's expand and prove the above.

$$10^4 = (10x10)x(10x10)=10000$$

Proof

$$1st\ 10^2 = 10 \times 10 = 100$$
$$2nd\ 10^2 = 10 \times 10 = 100$$
$$100 \times 100 = 10000$$

So adding the exponents is the same as multiplying, since the log of the number represents some exponent of a base ten number, its the same as multiplying the base number.
This is know as an exponential scale based on the root number 10

But before I go any further I will state that in our electronics we use logarithms as an expression of ratios comparing some gain or loss, the actual logarithm represents a ratio, in the above examples the ratio in the case of 10^2 that equals 100 and can be expressed as Log (100) this can also be represented as Log (100/1) One must note the implied reference that represented by some other term, that defines the comparison as the denominator. You will come across terms like dbi, dbm, dbu etc.
you have to understand, what the comparison is, in the case of "i" this is isotropic as the reference in stating antenna gain, in the case of "m" this is one millivolts, in the case of "u" this is one micro volts
So if I make a statement that the gain of an amplifier is 10dbm the gain is a ratio over one millivolt.

Converting a log figure to a real number
The numerical value of a **log** represents a power of 20; we can say this as a rule

$$log1 = 10^1$$
$$\text{As a ratio;} \qquad log1 = 10^{(1/1)}$$

So the conversion is a simple matter, take the log value and use it as the exponent of number 10, its really that simple. No need to use some online calculator.

Logarithm used as an expression of power

Power = V x I this value represents both the combined voltage and current
The power equation for db is 10Log(P1/P2)
So if I am stating the power of an amplifier as $10db_w$ the subscript (w) indicates my reference is one watt.
To find the power in watts we need to transpose the formula

$$10db = 10 \times Log (P/1Watt)$$

We need get rid of the "log" term

$$\text{Thus } 10/10 = \log (P/1)$$
$$1 = \log (P)$$
$$P=10^1$$
$$P=10\text{Watts}$$

Remember the reference was 1 watt

Gain is relative easy to convert but attenuation is a bit tricky, a negative power has to be converted to a positive number.

Lets consider the value -10db = 10 log (P/1)
We know that a positive log is a multiplication, thus s negative log is a division
we can define that Log $1 = 10^1$
And Log-1 = $1/ 10^1$ this is the inverse of the above.
Let me be more explicit with a real number, take 2, the inverse of 2 is 1/2 = .5, get the idea, the tricky part to understand that with linear numbers the relationship is not that clear, you could say why is the reciprocal of 2 equal to .5, its a fair question because two is twice one and .5 is half of 1, this is the key to understanding the reciprocal of a number. I also know that once a number goes in the negative direction of the number scale it becomes a negative number, so by taking the reciprocal of 2 the number becomes negative.
I know sounds a bit like smoke and mirrors.
We can see that the value I chose with the above example gives 10 time over 1Watt thus 10 db is 10 times that of one watt, its logical to say that -10 db will be 1/10 of 1 watt, that is 0.1 Watt

Lets see the maths for this calculation.

$$-10db = 10Log (P/1)$$

Rewriting the equation using reciprocals

$$1/-10 = 1/10Log (P/1)$$

Transpose terms and we end with

$$10/-10 = Log (P/1)$$
$$\text{Thus, } 1 = 1/ \text{Log } P$$

Taking P to the other side we have

$$P= 1/ 10^1$$
$$P= 1/10$$
$$P= 0.1 \text{ Watt}$$

One tenth of the reference, so the method is true.

From the above we can make this generality, if the Log is positive the conversion is 10^x if the Log is negative then its the reciprocal $1/10^x$

The simplest way is to deal with the solution as if it was a positive number, just take the reciprocal of the answer and that will be the solution of a negative db value.

Lets test this

$$-10db = 10Log\ (P/1)$$

$$P=10^1$$
$$P=10$$

The purpose of this article is to show how easy it is to convert, for more understanding at time another article helps, so here is a link to the ARRL explanation
http://www.arrl.org/files/file/Instructor%20resources/A%20Tutorial%20on%20the%20Dec-N0AX.pdf

Now I am not about to explain why volts and amp ratios the formula is 20Log
its to do that the terms for power ratio includes the ratios of both current and voltage

So remember to note the reference, if dealing with volts or current the equations are

$$db = 20\ Log(V1/V2) \quad and \quad db= 20\ Log(I1/I2)$$

The conversion method is still the same, if negative take the reciprocal of the answer.